



**MACSI-net**

A Roadmap for Mathematics  
in European Industry

## MATHEMATICS:

Key to the European Knowledge-based Economy

A Roadmap for Mathematics in European Industry

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# MACSI-net Roadmap, March 2004

Mathematics, Computing and Simulation for Industry

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## ★ EXECUTIVE SUMMARY

The European Union has set itself the goal of becoming the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion. This report outlines the vital role that mathematics has to play in achieving this goal.

Leading figures from industry and academia have contributed their views, through events organised by MACSI-net and through a series of special interviews, on the role that mathematics and computing are playing now and will play in the future in contributing to European competitiveness. These views form the basis for this report and clearly indicate the importance of mathematics and computing.

One of the main messages of this report is that mathematics should be seen as a technology in its own right. Furthermore, it is a very special technology in that new innovations in mathematics can often be applied to a wide range of areas with little modification, leading to further innovation.

Six technology themes, essentially mathematical in nature, have been identified as being of central importance to European Industry. The themes are: simulation of processes and products; optimization; control and design; uncertainty and risk; management and exploitation of data; virtual material design; and biotechnology, food and health. These themes encapsulate the areas in which mathematics is already being used in industry. They also present the mathematics community with broad and challenging areas where new advances will have a major impact.

One of the main drivers for the development of a knowledge-based economy is the computer revolution that is providing industry and commerce with ever more powerful machines at increasingly affordable costs. Mathematics is playing an essential role in this revolution in developing the algorithms and techniques that enable computers to be deployed to solve industrial problems. Europe needs a dynamic mathematics community interacting actively with industry and commerce on the one hand and the science base if it is to compete in the global market of the future where innovation will be the key to success.

The basic message of this report is that if Europe is to achieve its goal of becoming the leading knowledge-based economy in the world, mathematics has a vital role to play. In many industrial sectors the value of mathematics is already proven, in others its potential contribution to competitiveness is becoming apparent.

The following recommendations are aimed at strengthening mathematics, particularly the mathematics needed for the future success of the European economy.

## MACSI-NET ROADMAP

➤ Mathematics should be regarded as a technology in its own right. Its crucial role in many industrial problems requires the active participation of mathematicians. Truly multidisciplinary projects will benefit significantly from the involvement of mathematical modellers and this should be encouraged by future funding programmes.  
**Consideration should be given to making the participation of mathematicians in appropriate multidisciplinary projects a condition of project funding.**

➤ There is a need for positive action to promote the increased use of mathematics by European industry. The success of local initiatives where mathematicians are working on industrially relevant problems is clear evidence that they are already making a significant contribution to the development of the knowledge-based economy. However, more needs to be done to encourage companies, especially Small and Medium-sized Enterprises (SMEs), to make use of mathematics and mathematicians.  
**Consideration should be given to creating a programme funding projects that will enable companies, especially SMEs, to explore areas where mathematics can make a contribution to their improved competitiveness.**

➤ There is an urgent need for more training in the area of industrial mathematics. It is essential to attract bright students to this area and to convey the challenge and the excitement of solving practical problems.  
**Consideration should be given to specific funding for training programmes in industrial mathematics across Europe.**

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## ★ INTRODUCTION

The purpose of this document is to explore the role and significance of mathematics for European Industry over the next ten years, and to present a clear vision of how mathematics can contribute to European competitiveness. It should be seen in the general context of the strategic goal that the European Union set itself for the next decade at the Lisbon European Council in spring 2000, namely:

➔ **to become the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion.**

This document has been prepared by MACSI-net, a European network, involving both academia and industry, devoted to promoting the use of mathematical models, computing and simulation in industry. The next section gives a brief history of MACSI-net and further background information on its activities.

The document presents an analysis of European Industry's needs over the next ten years. This analysis was based on a variety of activities carried out by MACSI-net, comprising some 70 special events attended by approximately 1700 people, of whom 30% were from industry. In addition, to complement these MACSI-net activities, a series of over 20 interviews was conducted with senior figures from industry and commerce across Europe. These leaders were asked for their views on the role of mathematics and computing in their particular sector, the potential future impact of these disciplines and the major challenges they face where they feel mathematics and computing have a contribution to make to future competitiveness. Thus, the information concerning industrial requirements, on which this report is based, came from companies varying in size from Small and Medium-sized Enterprises (SMEs) to multinationals and from a wide range of industry sectors. Academics and industrialists from many European countries, including Eastern European and non-EU countries, have contributed to this report.

Technology may be defined as the practical application of science to the problems of commerce and industry. In the context of this report, mathematics is regarded as a technology in its own right – namely the practical application of the mathematical sciences to commerce and industry. Mathematics is a very special technology in that it applies to virtually every business area. Furthermore, mathematical ideas developed in one business area may often be applied to a completely different area with considerable success. This cross fertilization can be a powerful driver for innovation. Six technology themes, essentially mathematical in nature, have been identified as being of central importance to European Industry.

## 1 INTRODUCTION

The themes are: simulation of processes and products; optimization, control and design; uncertainty and risk; management and exploitation of data; virtual material design; biotechnology, food and health. These themes encapsulate the areas in which mathematics is already being used in industry. They also present the mathematics community with broad and challenging areas where new advances will have a major impact. The technology themes and the challenges they present are described in section 3. In addition to the six themes, a number of emerging themes have been identified and these are also described in section 3. Section 4 describes how mathematical technologies can be put to use, illustrated with six case studies drawn from a range of industries.

The findings reported here will be of interest to industrialists, policy makers, funding organisations and academics, all of whom have a major role to play in the future development of the European economy. Every attempt has been made to make the document accessible so that a wider audience can appreciate the importance and relevance of mathematics. Section 5 presents some conclusions together with a number of broad challenges facing Europe if it is to harness successfully the power of mathematics to the benefit of its citizens. Finally, section 6 contains some practical recommendations for what can be done to meet these challenges.

This document demonstrates the wide range of applications and the impact that mathematics and computing are having on the European economy. They are already making a vital contribution to European competitiveness and their role and importance will only increase in the future as the European Union seeks to achieve the goal it set itself at the Lisbon Council.

## 2 MACSI-NET

### ★ MACSI-NET

MACSI-net was set up as a European Network of Excellence, aiming to further the deployment of applied mathematics and computing to meet the needs of European industry. It was funded by the European Commission, in particular IST (Information Society Technologies). MACSI-net's activities started in October 2000, with the first stage finishing in April 2004. The major task was to charter the opportunities for co-operation between academia and industry and how this co-operation could be promoted in the area of applied mathematics and computing in particular. To achieve this MACSI-net has held Strategic Meetings with industry on a number of specific topics, organized Summer Courses and Industrial Days for industrialists, arranged visits of experts to companies, run courses on Modelling and last but not least carried out a series of interviews with leading figures in European Industry.

During the first stage of its existence, MACSI-net concentrated on seven major industrial areas: namely aeronautics, material processes, chemistry, environment, bioengineering and medicine, telecommunication and energy. The network has a matrix structure, in which the application areas are to be seen as columns and the rows as mathematical academic disciplines. The activities were aimed at trying to cover as much of this matrix as possible. Naturally, the universality of mathematics makes techniques useful in one application area often useful (with some adaptations) in another. Hence there has been much cross-fertilisation.

The members of MACSI-net are European institutions with an established record of links to industry as well as a high academic profile. During its first period of operation MACSI-net acquired some 500 more so-called associated nodes. These members and nodes cover most of Europe including Eastern Europe, with a clear bias to the former. A large percentage of the nodes are from industry (30%). Moreover, there is a good spread of academic disciplines and application areas. The research activities of MACSI-net are organised through working groups, each one devoted to a

particular theme. Most of the 17 working groups have been very active and have organized series of events ranging from meetings to encounters with industry, through workshops on specific topics to the formation of activity groups. Some of these activities have already lead to funding for larger co-operative projects. The application areas covered by these working groups included acoustics, glass, finance, insurance, porous materials, medicine, telecommunication, paper manufacturing and cardiovascular functions. The mathematical disciplines included optimization, inverse problems, model reduction, parallel computing and CAD and geometric modelling.

An executive committee, chaired by the co-ordinator, and a strategy board, which has responsibility for long term planning of the group's activities, control these working groups. The executive committee consists of representatives from the members of the working group, whereas the strategy board is composed of industrialists and senior figures from academia in equal numbers.

The network is running a variety of meetings; the outcome of each meeting is reported to the executive committee. The network as a whole had larger kick-off and mid-term meetings. The events organised by the various working groups took place all over Europe - in total more than 70 - attracting about 1700 experts from industry and universities. Some of these meetings had the format of an industrial day, where general problems from industry were discussed; some had the form of a week-long modelling meeting, where problems were partially solved to demonstrate the usefulness of mathematical approaches; finally, some were in the form of (summer) schools to show young researchers how interesting industrial problems can be and how they may be successfully tackled. The participants were often asked to complete questionnaires to be used to monitor the success of the meeting and to identify possible other relevant areas. Special mention should also be made of visits to industry. Besides the large number of people from industry participating in the various workshops and events (on average 30%, some meetings had more than 70%), visits were made to companies. Often these took place in the context of a research activity within a working group.

A substantial number of visits were planned to complement the findings of the working groups. It was decided to interview leading industrial managers in a number of European industries to have their vision about the need and use of mathematics in industry in Europe in the coming years. Their vision, in addition to the overall result of the activities of the working groups, forms an essential part of this road map.

## 3 TECHNOLOGY THEMES

### ★ TECHNOLOGY THEMES

The information acquired during the three and half years of the MACSI-net project together with the interviews with senior figures from industry was carefully analysed and six technology themes clearly emerged as being of central importance to industry.

The themes are:

**simulation of processes and products;  
optimization, control and design;  
uncertainty and risk;  
management and exploitation of data;  
virtual material design;  
biotechnology, food and health.**

The themes encapsulate the mathematical challenges facing many sectors of the European economy and some apply to almost all sectors. In the following sections, the content of each theme is explored. Each section begins with a brief, but general description of the theme, followed by examples that show how the theme is relevant to various industries. The sections end with a summary of some of the mathematical challenges of importance to industry.

Two issues of major importance to the European economy are safety and efficiency, for obvious reasons. These issues occur repeatedly in the following sections and progress in the six technology themes will have a significant impact on them.

Historically, simulation has been the most important theme across Europe, and has had the greatest industrial impact. It is certainly the area in which MACSI-net has been most active. Its importance is expected to continue, and indeed increase, and the other themes are growing in significance. In the vision of the future that has emerged, simulation will continue to play a vital role, with the other five themes becoming increasingly important for an even wider range of industrial sectors.



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SIMULATION

## II Simulation of processes and products

Simulation is the process of imitating some aspect of the real world using a mathematical 'model'. The model represents the real-world phenomenon, and, by studying it with analytical or numerical techniques, both qualitative and quantitative predictions can be made. The reliability of the predictions depends on how good the original model is and how effective the solutions methods are. Models for complex, or poorly understood, phenomena may not capture the essential features of the phenomena. Complicated models may be too difficult to solve with existing mathematical techniques, either analytical or numerical.

Despite these difficulties, simulation has become firmly established as one of the major pillars of modern science and engineering, complementing the traditional experimental and theoretical approaches. The main reason for this is the remarkable advances in computer technology, resulting in the widespread availability of increasingly powerful computers.

Simulation is now routinely used in many parts of European industry to support, and sometimes replace, experimentation. It can have a dramatic effect on the design process, reducing the need for costly prototypes and increasing the speed with which new products can be brought to market. Simulation is also very effective in safety and reliability studies and in managing and extending the lifetime of existing assets.

In some industries, such as the **aerospace** and **automotive** industries, simulation is already a vital and well-established part of the product development cycle. In others, it is only just beginning to be used, but there is little doubt that in the future it will be increasingly important.

★ In the **aerospace** sector, simulation is widely used in all aspects of aircraft design. It is used to help design jet engines that are more efficient so that they use less fuel, produce fewer harmful emissions and make less noise. Simulation plays, for example, an important role in assessing the impact of bird strikes on engines by reducing the number of costly experiments that have to be carried out.

Computational aerodynamics is used to understand the air flow over wings and fuselage and is vital to the performance of aircraft and also to their safety against icing, for example. Computational electromagnetics is used to make sure that the electrical equipment in an aircraft functions correctly in the presence of electromagnetic fields and to study how an aircraft might be affected by a

lightening strike. It is also used to help design ships and aircraft so that they are harder to detect using radar, which is important for the **defence** industry.

The **aerospace** sector is always seeking new materials capable of withstanding large forces and extremes of temperature, and simulation is used in the development of these materials.

★ Simulation has had a dramatic impact in the **automotive** sector in recent years, affecting virtually every aspect of the design and manufacture of road vehicles.

Stamped components, made by stamping a single sheet of metal, are becoming more complex leading to fewer welded joints, simpler assembly lines and reduced costs. Approximately 75% of the world's stamped metal is used by the automotive sector. Simulation is used to design the machines that make the stamped components and can reduce significantly the time taken to produce a prototype machine while increasing the chances of it working first time.

Simulation is used in chassis and suspension design to improve comfort and handling. Fatigue analysis is used to assess the reliability of various car components.

Increasingly sophisticated electronic systems are being deployed in cars, and computational electromagnetics is used, in much the same way as in the aerospace industry, to ensure that these systems function correctly in the presence of electromagnetic fields.

Safety is a major concern of the **automotive** sector and crash and airbag simulation, using computer models, are well-established techniques used by design engineers.

In other areas, such as the combustion processes in engines and climate control, simulation technologies are being actively developed, but are not yet widely deployed.

Several manufacturers are currently developing the concept of the Digital Factory, in which every aspect of the manufacturing process will be optimized and managed using sophisticated simulation tools.

★ Aerodynamic considerations are increasingly important for the **transport** sector at large in the design of cars, lorries, buses and trains. Controlling the noise generated by all types of vehicles can improve passenger comfort and minimise noise pollution of the environment. Simulation is used in all these areas.

- ★ The **chemical** industry uses computational fluid dynamics to help design various forms of moulding and extrusion equipment for producing polymer-based products.
- ★ In the **construction** industry computer analysis is routinely used in the design of new buildings to ensure their mechanical integrity and ability to withstand environmental loading caused by wind, snow and, in vulnerable regions, earthquake. This type of analysis is especially important if the design or method of construction is particularly innovative.  
Fire safety is very important in the **construction** industry and computational fluid dynamics is used to assess the safety of new buildings and tunnels, for example.  
One of major challenges in the civil engineering field is the safety assessment of large structures such as tunnels, dams, chimneys and cooling towers associated with power plants. As these structures approach the limits of their design life, it is necessary to decide whether to decommission and replace them or to refurbish them. A range of technical and economic issues, as well as safety considerations, influences these decisions and computer simulation is becoming an increasingly important tool for the civil engineer in this context.
- ★ The **oil and gas** industry has been using simulation for some time to predict the flow of oil and gas in underground reservoirs in order to manage and maximize the amount of the resource extracted. These techniques will continue to be very important as hydrocarbon resources are developed from increasingly difficult geological formations under hostile operating conditions.  
Simulation is also used to understand how oil and gas flow through the very long pipelines that are often needed to transport them.
- ★ In the **entertainment** industry, the objective of computer games is to create a highly interactive, high-definition, audio-visual consumer experience. The realism of the simulation is based on sophisticated mathematical models. In addition, the simulations must be generated in real-time and often with limited computational resources. In the future, this technology will be used to deliver a broader range of content to a wider audience.
- ★ Simulation of electromagnetic waves, through cluttered environments and in various weather conditions, is used in the **telecommunications** sector in the design of mobile telephones and to improve the management of the radio spectrum.
- ★ In the **electronics** industry, many problems associated with the design of integrated circuits (IC) can only be addressed with the help of simulation. The trends here are to make devices smaller and faster, while at the same time providing greater functionality. For mobile devices, it is also important to reduce power consumption. These trends mean that increasingly accurate models of transistor behaviour are required. Furthermore, effects that previously could either be ignored or else treated very simply, such as leakage currents and the effect of connections, which lead to cross-talk, now have to be modelled in more detail.
- ★ In the **metals** industry, there is a continual drive to improve the efficiency of plant and the quality of the finished product. In many cases, this can only be achieved by simulating the basic processes involved. This is because the operating environment in such plants is too hostile to allow extensive instrumentation.  
Simulation is used to help design moulds for casting processes. Good designs can result in lower costs, a reduction in the number of bad casts and an extension of the lifetime of the casting equipment.

- ★ The **nuclear power** industry has been using simulation for many years in the design of new plant, and to support the continued safe operation of existing plant. Many physical processes take place inside nuclear power stations including fluid flow and heat transfer, thermal stressing of components, neutron transport, nuclear reactions, radiation induced changes in materials and so on. A good understanding of all these processes is required and simulation is an invaluable tool in developing such understanding. Safety is a major preoccupation in the nuclear industry, and here simulation plays an important role. It is used to help design and interpret experiments and also to complement experimental work, enabling the impact of hypothetical severe accidents to be investigated. Simulation is used extensively in the investigation of options for the long-term disposal of radioactive wastes. The timescales involved are so long that modelling is an essential part of the performance assessment of a potential repository for radioactive waste.
- ★ The design and construction of **coastal defences** and **offshore** structures present many engineering problems and simulation is extensively used. Offshore structures have to withstand extreme sea conditions and often operate for 20 years or more. Harbours and sea defences have to withstand large breaking waves. The interactions between a structure and the waves and, for a fixed structure, between the foundations and the seabed, must be understood. The effects of fatigue on the reliability of the structure must be assessed to ensure safe operation.
- ★ Computational fluid mechanics is used in the **maritime** industry in the design of both naval and merchant vessels. Moreover, it is an important tool in designing sea defences.

It is clear that the scope of simulation within European industry is very wide indeed. Nevertheless, the industry's main needs, and the associated mathematical challenges may be summarised as follows.

Increasingly, simulation is used, not only to supplement experimentation and prototyping, but also to replace them. Consequently, there is a growing need for greater physical realism. Some simulations lack sufficient realism because good models do not exist, or because model data are uncertain or incomplete, or because the problem is computationally intractable.

- ⇒ **The challenge is to develop improved models, capable of representing the phenomenon in question with an adequate and known degree of accuracy.**

Improving the realism of a simulation usually entails an associated increase in the length of time required to carry out the simulation. In many circumstances, there is a need for a rapid response to changes in the model parameters. The need for efficiency is particularly important in the entertainment industry, because the simulations must be carried out in real time on commodity hardware and must not affect the visual experience associated with a computer game. However, virtually all areas of industry would benefit enormously from faster simulations.

- ⇒ **The challenge is to develop more efficient and robust algorithms for solving the mathematical models that arise in simulation, making effective use of new computer hardware.**

Many problems that arise in industry have a wide range of length and time scales. For example, changes at the molecular level may influence the properties of a material, which affects its macroscopic behaviour. It is often not possible to employ a fine-scale model for the whole of the simulation, and so multiscale models are required. This is already an active area of research and

- ⇒ **the challenge is to develop techniques that may be applied reliably and effectively to multiscale problems arising in industry.**



MAKING SURE THAT SHOPS HAVE THE APPROPRIATE STOCKS

## III Optimization, control and design

While simulation is very important in understanding an industrial process or product, what is often required by industry is a design that meets certain objectives. So, rather than asking how a product performs, the question is, how should the product be designed so that it performs in a specified way? Optimization is the mathematical discipline that addresses these questions. Scheduling, planning and logistics also fall within the area of optimization. Optimal control is used to provide real-time control of an industrial process or a product, such as a plane or a car, in response to current operating conditions. A related area is that of inverse problems, where the parameters for a model must be estimated from measurements of the model output. The case study in the next section contains more information on inverse problems.

- ★ In the **aerospace** industry, the shape of a commercial aircraft's wings has to be chosen to reduce drag whilst ensuring stability during flight. For a military aircraft speed and manoeuvrability might be the design criteria. Airframes must be as light as possible whilst being strong enough to withstand the forces the aircraft experiences during operation. Modern aircraft employ sophisticated electronic systems and instrumentation, and control systems that are implemented in software. Modern commercial jet engines are designed to minimize their impact on the environment, through emissions and noise, whilst maximizing fuel economy and performance.
- ★ In the **automotive** sector, designing a new car is almost as complex a task as designing a new plane. The car chassis and body have to provide good handling and performance, and take account of passenger comfort and safety at the same time as meeting aesthetic requirements. Car engines have to be designed to be efficient and keep harmful exhaust emissions within levels set by the regulators. The use of electronic engine and chassis control and information and communication systems is now common, and these systems must be electromagnetically compatible. In addition, since most cars are mass-produced, the factory and assembly line must be designed to be efficient and cost effective.
- ★ In the **construction** industry, new modern buildings have to be designed so that they are safe and able to withstand the various loads to which they will be subject. Fire safety regulations must be met, and the environmental impact of the building must, increasingly, be considered. So issues such as energy consumption, heating, lighting and ventilation must all be addressed.
- ★ In the **telecommunications** sector, designing a mobile phone requires many disciplines and technologies. These include microelectronics, software engineering, data compression, network design and signal processing. Phones have to be easy to use, and, increasingly, aesthetically pleasing, while providing the desired functionality. The trend in mobile telephony towards personal multimedia communication and integration with the Internet provides significant challenges for the design teams.

- ★ In the **retail and distribution** and **transport** sectors it is important to minimize transport and storage costs, whilst making sure that goods are delivered on time and shops have the appropriate stocks. Generally managing the supply chain in an efficient manner is a complicated area where optimization plays a vital role.
- ★ In the **finance** industry, managing large portfolios in the presence of transaction costs in an optimal way requires the solution of complex optimization problems.
- ★ Deregulation in the **electricity supply** industry in some parts of Europe means that the electricity generators and end-users operate in an extremely fluid market where contracts for the sale and purchase of electricity are exchanged on time-scales from months to half-hours. The electricity system must be designed to accommodate these fluctuations in the market, as well as the longer-term energy and environmental influences on the market, in an efficient manner. Balancing supply and demand in the electricity system is a difficult problem requiring sophisticated control mechanisms to ensure continuity and quality of the electricity supply.
- ★ The detection of defects and cracks in structures is important in many industrial sectors and is particularly important as far as safety is concerned in the **transport** sector and in the **chemical** and **nuclear** industries. Non destructive testing techniques are widely used and these are based on the mathematics of inverse problems.

Optimization, control and design, by their very nature, have a very wide scope. Nevertheless, certain common challenges can be identified. They are:

Combinatorial optimization problems occur in many applications, where they are often key to identifying optimal strategies. Optimal solutions to this type of problem are very hard to obtain in general, and a practical goal is a good approximate strategy rather than an optimal one.

⇒ **The challenge is to develop fast algorithms that give near optimal results for a wide range of problems.**

Inverse problems occur in many areas of industry. The value of analysing such problems depends critically on how fast the analyses can be carried out.

⇒ **The challenge is to develop fast algorithms for the real time identification of parameters in realistic three-dimensional models governed by partial differential equations and this requires the efficient coupling of fast solvers for such equations with robust iterative methods for inverse problems.**





## II Uncertainty and risk

Uncertainty arises when there is not enough information to allow accurate predictions about a system or process to be made. Risk results from a combination of uncertainty and exposure to loss. The loss may take many forms, such as adverse consequences for human health and life, damage to property, reduction in profits, or detrimental impact on the environment. Understanding the impact of uncertainty, and quantifying and assessing risk are important issues for most industry sectors.

- ★ Uncertainty arises from many sources in the **manufacturing** industries. Examples are human factors, variability in raw materials and process variability. Understanding and controlling the impact of uncertainty on the quality of the final product is very important.
- ★ Uncertainties in financial markets, interest rates, foreign exchange rates, and the prices of basic commodities often provide business opportunities for the **banking and finance** sector. Financial derivatives, based on some underlying asset, are important tools in this sector and agreeing a fair price for such derivatives is vital. This task is essentially mathematical in nature and often involves computer simulation. A good understanding of the uncertainties and risks associated with the provision of financial services is vital to the economic success of the **banking and finance** sector.
- ★ The **insurance** industry is primarily concerned with managing uncertainty and risk on behalf of its clients. A clear and quantifiable measure of the costs associated with risk is essential for this industry to function properly. Insurance companies use sophisticated mathematical models both to price their products and to manage the financial risks to which the company itself is exposed. In this regard, hazards with large consequences are of critical importance. In some circumstances, the quantitative assessment of insurance risk depends on accurate modelling of unusual or extreme events, for example extreme weather conditions, flooding and other natural disasters. To model these extreme events it is necessary to understand how the low probabilities with which they occur behave. Mathematics and computer simulation are now indispensable in the **insurance** industry in general and especially in the reinsurance industry, and in the future, there is likely to be an even greater need for these tools.
- ★ Forecasting in the presence of uncertainty is crucial for profitability in the **retail and distribution** sector. For example, too much stock leads to wastage and too little leads to customer dissatisfaction, both of which have a direct impact on profitability. Launching and promoting new products are important factors in retaining customer loyalty in a very competitive environment. Such launches are subject to risks and uncertainties that have to be managed.
- ★ Planning for uncertain future capacity is a major challenge for the **transport** sector for reasons similar to those in **retail and distribution**, with the added complication that safety is an important factor. Economic conditions have a large impact on this sector, so understanding future economic trends is very important for the planning process.

- ★ Planning future capacity for the **electricity** industry has always been subject to uncertainty. In the past, a conservative approach, in which continuity of supply has been of paramount concern, has generally been adopted. Increased competition, as the industry is deregulated across Europe, will make the management of uncertainty and risk even more important in the future. Maintaining security of supply in an increasingly competitive and uncertain market will be a major challenge.
- ★ Futures and options are potentially important instruments for the **electricity supply** industry in managing risk in the market. Enron were pioneering sophisticated options involving energy, weather derivatives and climate change quotas. Since the collapse of Enron, this approach does not appear to have been actively pursued. However, it is a natural and powerful development and is likely to be used in the future in this area.
- ★ Safety is of paramount importance in many industries, including the **transport, nuclear, construction** and **chemical** industries. The long-term impact of industrial operations on the environment is also of major concern. Consequently, considerable importance is attached to assessing, quantifying and reducing the risks involved. The major financial implications of serious incidents mean that companies pay careful attention to safety and environmental issues and the associated risks when making investment decisions.

One of the most remarkable achievements of mathematics has been the development of techniques that allow risk and uncertainty to be treated in a quantitative manner, and, to some extent, controlled. The range of problems to which these techniques can be applied is increasing with a consequent beneficial impact on the performance of the economy generally. Uncertainty and risk present a number of key mathematical challenges, which include the following.

One widely-used approach to treating uncertainty is based on stochastic models. Such models always contain parameters, and there is the challenge of estimating the values of these parameters from the available data, which may be biased, noisy and incomplete. In many situations an estimate of the likely errors in the parameters is also needed. For example, in the insurance industry models of complex risks, such as natural catastrophes, contain parameters that must be estimated from data on past occurrences of such events.

➤ **The challenge is to develop robust and reliable methods for estimating parameters in increasingly complex stochastic models.**

Incorporating models of uncertainty within existing mathematical models inevitably leads to an increase in mathematical difficulty. However, this increase in difficulty means that a wider range of questions can be answered with such models. For example, in a safety study it may be possible to demonstrate that a certain event occurs only with very low probability. Existing methods of analysis may not be sufficiently powerful to solve the new models. In the field of financial mathematics, the analysis of more complex financial instruments leads to mathematical problems in higher dimensions that can be intractable with existing numerical methods.

⇒ **The challenge is to develop more powerful techniques, both analytical and computational, for analysing stochastic models.**

Human behaviour is perhaps the greatest source of uncertainty confronting industry. The influence of human factors on the design and marketing of new products is a major issue in most sectors. Human behaviour often plays a significant role in safety issues.

⇒ **The challenge is to develop appropriate, quantitative models of human behaviour with known scope of applicability.**



PROCESSING DATA

## III Management and exploitation of data

The significant advances in computer systems for collecting, storing, retrieving and processing digital data coupled with breakthroughs in sensor technology have led to unprecedented volumes of data being generated and stored. The management and effective use of these data sets is both a challenge and an opportunity for industry. In society generally, more and more data are being stored in digital format and the products of some industries are delivered in the form of digital data sets. Issues associated with the use and management of digital data will become increasingly important in the knowledge-based economy of the future.

Understanding information flow in large companies and in the economy in general is an important challenge that has a significant mathematical component as well as the human element. The automatic recognition and processing of patterns in images, sounds and signals already has many applications, and the potential for future applications, as techniques improve, is enormous. Data mining, in the sense of extracting information and knowledge from large data sets, is another important area with huge potential. Using data to build models where the underlying phenomena are not fully understood, such as in economics and medicine, is an increasingly important area of research. Models based purely on the data are often called “black-box” models, whereas models based on a combination of data and models of the underlying phenomena are called “grey-box” models.

Many important issues related to data and its use emerged during this study.

- ★ In the **banking, insurance and finance** sector, data on both customers and on the financial markets are collected in huge volumes. Good quality data in sufficient volume is a key element in developing more powerful risk management tools for the **insurance** industry.
- ★ Companies in the **retail and distribution** sector generate and process large data sets. For example, the data may come from loyalty card schemes and can be used to understand customers’ needs and shopping patterns. This information can then be used, for example, to help plan store layout and stocking policies to meet customers’ expectations, thereby encouraging loyalty.
- ★ **Transportation and warehousing** companies have to manage large data sets containing information on stocks, shipments, transport costs, customers’ needs etc. A crucial element for success in this area is to be able to optimize the available resources, subject to a wide variety of constraints.
- ★ In the **oil and gas** industry, companies generate and process large volumes of data during the course of geological characterisation of potential reservoirs. Production facilities, such as boreholes, are increasingly being equipped with sophisticated sensors, and the data collected can be used to improve the management of the resource.
- ★ In the **telecommunications** sector, data are collected on customer behaviour and this influences new product developments. The increased functionality of mobile communications and computing devices requires reliable and efficient means of transferring large volumes of data. Issues of data compression, signal processing and security are very important in this context. Innovative ways of using the data generated by mobile devices are being developed. For example, in Germany, information on the location of mobile telephones is being used in a trial traffic management scheme.
- ★ In the **rail industry**, data on the state of the rail infrastructure can be collected from devices fitted to ordinary passenger trains.

- ★ It is increasingly common in the **manufacturing** sector for companies to collect data from production lines as part of routine operations, although these data are often not exploited systematically. In the **automotive** sector some companies are developing systems to manage the entire production and manufacturing process and here it is the organisation of the whole data flow and the interaction of the sub-processes that is the real challenge.
- ★ Fingerprint authentication technology is used to provide physical access control systems in the **security** industry. Intelligent surveillance systems, based on smart camera technology with capabilities for face recognition, for example, are also being developed and deployed. Systems for processing handwriting and transferring it to digital media have many applications in the **security** industry.
- ★ Products for scanning, processing and storing printed text are based on image processing techniques, and are becoming increasingly important in the modern, knowledge based economy for **all** sectors. These systems can be very sophisticated, with some forms of content recognition now becoming possible.
- ★ The **entertainment** industry makes extensive use of image processing techniques. The use of digitally generated special effects in the **film** industry is a very well known example. Large volumes of data are used in these areas. The advent of the digital age has given rise to new legal and technical issues, such as the protection of legal rights over content. Video piracy, for example, costs the **film** industry large sums of money, despite the fact that the sixth or seventh copy becomes almost unusable. Digital media, on the other hand, suffers little or no degradation on copying. These problems and their solutions are essentially mathematical in nature.
- ★ **Governments** collect and analyse large volumes of data. This information is used to plan economic and other policies and to inform the general public – an essential function in a democratic society.
- ★ The **media** industry has to manage large amounts of data and this is likely to increase dramatically in the future, as digital TV and radio become the norm and ‘on demand’ services, such as films over the Internet, become more widely available. The large **broadcasting** companies in Europe already make use of products that aim to integrate all the media in an enterprise into an online, media management system, providing access to all who need it in the company. There are many technical challenges arising from different formats and different protocols, and providing facilities for content, rather than just data, extraction.
- ★ The extraction of ‘content’ from a variety of different media is a common issue for a range of information-intensive enterprises. Examples include **libraries, healthcare organisations,** and the **security** industry.

In summary, huge data sets, often running into terabytes<sup>1</sup>, are now collected routinely across a wide range of industrial sectors. One of the primary concerns of the information technology industry is managing the efficient and reliable storage and retrieval of such data. Making effective use of such data is an endeavour in which mathematics plays a central and vital role. Some of the many mathematical challenges include the following.

Advances in algorithms, coupled with the continued growth in the power of computers, offer new possibilities for visualizing, analysing and otherwise “mining” data. Nevertheless, these techniques are only able to address in part the problem of turning data into information and knowledge. There is a clear need for techniques that greatly extend the scope of this activity - what might be called “cognitive information processing”.

- ⇒ **The challenge is to develop techniques that can process data to provide information and even knowledge in a form that can be commercially exploited.**

Despite the huge growth in systems and techniques for collecting data, there are still limitations in sensor technology and other forms of data collection that can adversely affect data quality. This presents mathematical challenges for the analysis and the use of such data in mathematical models.

- ⇒ **The challenge is to develop improved methods for taking into account data quality and its impact on the models.**

Many products and services are delivered in the form of digital data sets (e.g. films on DVD, music on CD, and the output of all software companies, via CD, tape, or direct download from the Internet) and even more will do so in the future. This gives rise to issues of security, privacy, ownership and access to digital data.

- ⇒ **The challenge is to develop the tools that will enable business to function efficiently and securely in the knowledge-based and data-rich economy of the future.**

One way of exploiting data is to use it to build models of systems that can then be used to understand the systems or to make predictions as to how they will behave. Such models may lead to a more fundamental understanding of the processes being examined. This area of mathematics is known as systems theory.

- ⇒ **The challenge is to develop general methods for producing black and grey box models that can be applied to data collected by industry to enable it to be exploited effectively.**

<sup>1</sup> 1 terabyte = 1,000,000,000,000 bytes



TEXTURE, CONSISTENCY AND FLAVOUR OF PRODUCTS

## III Virtual material design

Understanding how materials behave is an important prerequisite for their use in industrial products. The study of the properties and uses of materials is the domain of materials science, and nowadays mathematics plays a very important role in this subject. One of the objectives of materials science is to design new materials that have desirable properties. This is called virtual material design. Mathematics and computer simulations are used to relate the large-scale (macroscopic) properties of materials, such as stiffness, strength, fatigue or wear, to the small-scale (microscopic) structure of the material. The microstructure is optimized to produce the required macroscopic properties. The microscale may be made up of a mixture of simpler materials, as in a composite material, or it may have to be modelled at the molecular level.

- ★ The **aerospace** industry is always searching for new materials that are stronger, lighter and more durable for obvious reasons. Improved material properties lead to significant commercial and safety benefits. An example is the use of turbine blades produced from single crystals. These are much stronger, for a given weight, than their traditionally produced counterparts.
- ★ When a component is periodically loaded and unloaded, energy is dissipated in the material and may lead to internal fractures and eventually to failure of the component. Understanding this process and being able to detect when a component is in danger of failing is very important in many industrial sectors, but particularly in the **transport** sector where safety is a major concern.
- ★ In the **paper** industry, simulation is used to understand the flow of water and wood pulp through a papermaking machine. In particular, the dilute paper stock flows rapidly through a filter, a flow box, a press and finally a drying cylinder. Understanding the nature of these flows can help to improve the design of the process.
- ★ Texture, consistency and flavour are important characteristics of products in the **food and drink** sector. They are properties of complex mixtures that must be carefully designed to achieve the desired effect.
- ★ In the **chemicals** industry, the design of new coatings that they are easy to apply, have the correct performance characteristics and are dura-

ble requires understanding that can be provided by various simulation techniques. The design and production of polymers is another area of importance to the **chemicals** industry.

- ★ In the **metals** industry, casting processes in steel production plants, and continuous casting of rolled steel determine the quality of the end product. It is important to understand how the process affects the microstructure of the metal and how that, in turn, affects the properties of the steel.

Materials science has become a very mathematical subject. Sophisticated and often difficult mathematical techniques, such as homogenisation, are used in order to understand the material properties. There are many mathematical challenges in this area.

The microstructure of a composite material can be very complicated, consisting of fibres, rods and sponges so that very complicated geometrical structures are involved. Modelling the microstructure may involve the use of continuum models or molecular dynamical simulations. It may even be necessary to include quantum mechanical effects in the microstructure modelling.

- **The challenge is to develop robust and reliable methods for treating the microstructures that arise in advanced materials.**

Calculating the bulk properties of complex materials requires a very wide range of length scales to be represented, from the microscale to the macroscale. Mathematical techniques, such as homogenisation, hierarchical multiscale methods and the quasi continuum method, are actively being developed to tackle these problems.

- **The challenge is to develop robust and reliable methods for dealing with the wide range of multiscale problems that arise in modelling new and complex materials.**

The ultimate goal in this area is to be able to design a new material with specified properties using mathematical and computational tools. This problem is sometimes called reverse engineering and is generally very difficult.

- **The challenge is to develop tools capable of computing the microstructure of a material so that it has the required macroscopic properties.**